

Q:- 1

Sol:- $y=e^{(\cot^{-1}x^4)^7}$

$$Dy/dx=(\cot^{-1}x^4)^7 e^{(\cot^{-1}x^4)^7} dy/dx(\cot^{-1}x^4)^7$$

$$=(\cot^{-1}x^4)^7 e^{(\cot^{-1}x^4)^7} (\cot^{-1}x^4)^6 dy/dx (\cot^{-1}x^4)$$

$$=7(\cot^{-1}x^4)^7 e^{(\cot^{-1}x^4)^7} (\cot^{-1}x^4)^6 \cdot \frac{1}{1+x^8} \cdot 4x^3$$

$$=-28x^3/(1+x^8)(\cot^{-1}x^4)^{13} e^{(\cot^{-1}x^4)^7} \dots\dots$$

Q2:-

Ans:- divide num, & denominator by x^2

$$d/x \rightarrow \infty \quad x^2[39+7/x]/x^2[54+19/x^2]$$

$$=39+7/\infty/54+19/\infty=39+0/54+0$$

$$=13/18 \text{ ans....}$$

Q3:

$$\text{Sol:- if } f(x) = \begin{cases} 12 & \text{if } x \text{ is an integer} \\ -7 & \text{if } x \text{ is a real no. which is not an integer} \end{cases}$$

Sol:

This function is discontinuous at $x=9$ because it takes value 12 if x is an integer

It takes value -7 if x is a real number which is not an integer

So it is not continuous.

Q4:-

$$\text{Ans:- } f(x) = 12x^6 - 4x^4 - 1$$

$$f'(x) = 72x^5 - 16x^3 + 45x^2$$

For maxima & minima, we have

$$f'(x) = 0$$

$$72x^5 - 16x^3 + 45x^2 = 0$$

$$x^2[72x^3 - 16x + 45] = 0$$

For minima,

$$F''(x) = 360x^4 - 48x^2 + 90x > 0$$

$$3x(120x^3 - 16x + 30) > 0$$

$$6x[60x^3 - 8x + 15] > 0$$

For maxima

$$F''(x) < 0$$

Q5:

$$\text{Sol:- } 3x^3 - 5x^2 + 2x + 5 = 0$$

$$a + \beta + \gamma = 5/3 \text{-----(1)}$$

$$a\beta + \beta\gamma + a\gamma = 2/3 \text{-----(2)}$$

$$a\beta\gamma = -5/3 \text{-----(3)}$$

now we have to find

$$a^2 + \beta^2 + a\beta, \beta^2 + \gamma^2 + \beta\gamma, a^2 + \gamma^2 + a\gamma$$

adding

$$a^2 + \beta^2 + a\beta, \beta^2 + \gamma^2 + \beta\gamma, a^2 + \gamma^2 + a\gamma$$

$$2(a^2 + \beta^2 + \gamma^2) + a\beta + \beta\gamma + a\gamma$$

$$2(a^2 + \beta^2 + \gamma^2) + 2/3 = 5/3$$

$$2(a^2 + \beta^2 + \gamma^2) = 1$$

$$A^2 + \beta^2 + \gamma^2 = 1/2$$

Q6:-

Sol:-

$$\int \tan^7 x \, dx$$

$$= \int \tan^6 x \cdot 1 \, dx \quad \text{integration by parts}$$

$$= [x - \tan^{-1}x] - \int \frac{1}{1+x^2} dx$$

$$= x - \tan^{-1}x - \frac{1}{2} \ln \frac{1+x}{1-x} + c$$

$x - \tan^{-1}x - \frac{1}{2} \ln \frac{1+x}{1-x} + c$ where c is a constant.

Q 7:-

Ans:- $r \cos \theta = 3$.

$$r \sin \theta = 4$$

$$r^2 = 9 + 16 = 25$$

$$r = 5$$

$$\tan \theta = \frac{4}{3}, \theta = \tan^{-1} \frac{4}{3}$$

n th root

$$z = (r \cos \theta + r i \sin \theta)$$

$$3 + 4i = r [\cos \theta + 2k\pi/n + i \sin(\theta + 2k\pi/n)]$$

$$3 + 4i = r \cos \theta + 2k\pi/n + i r \sin \theta + 2k\pi/n$$

Put $k = [-4, -2, -1, 0, 1, 2, 3]$

Q:8

Area bounded by the curve $y = e^{2x}$, the coordinates $x=4$, & $x=7$

$$\int_a^b y \, dx = \int_4^7 e^{2x} \, dx$$

$$= \frac{e^{2x}}{2} \Big|_4^7 = \frac{e^{14} - e^8}{2}$$

$$= \frac{e^{14}}{2} - \frac{e^8}{2}$$

Q9:

(a) Sol: equation of the line joining two points

$$(-7, 6, -3) \text{ \& } (7, -6, 3)$$

$$\frac{x+7}{14} = \frac{y-6}{-12}$$

$$= \frac{z+3}{6}$$

(b) Sol: any sphere is given

$$x^2+y^2+z^2-18+k(3x+3y+3z-11)=0, \quad k \in \mathbb{R}$$

Its center = $(-3k/2, -3k/2, -3k/2)$

Its distance from origin ie: $(0,0,0)$

